Phenomenology of the Standard Model from Lattice QCD[1]

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Abstract

Some recent results of lattice QCD calculations which are relevant for the phenomenology of the Standard Model are reviewed. They concern the lattice determinations of quark masses, studies of $K - \bar{K}$ and $B - \bar{B}$ mixings, and a prediction of the B_s^0-mesons lifetime difference. The results of a recent analysis of the CKM unitarity triangle, which is mostly based on the lattice calculations of the relevant hadronic matrix elements, are also presented.

1 Introduction

An accurate determination of the Standard Model free parameters in the quark sector, namely quark masses and CKM mixing angles, is a task of fundamental importance facing both experimentalist and theoretical particle physicist. Among these parameters, for instance, the charm and bottom quark masses enter through the heavy quark expansion the theoretical expressions of several cross sections and decay rates. The elements of the CKM mixing matrix and the angles of the unitarity triangle control the intensity of hadron weak decays and the mixing amplitudes of K and B mesons. The area of this triangle defines the extent of CP-violation in the Standard Model. On the experimental side, a major advance in this context has been represented by the recent determination of the parameter ε'/ε [1, 2], which controls direct CP-violation in $K \rightarrow \pi\pi$ decays. This measurement provides evidence of a CKM origin of CP-violation, and the evaluation of ε'/ε within the Standard Model, or some of its proposed extensions, still represents a challenging theoretical task. From a more theoretical point of view, an accurate determination of quark masses and mixing angles may give insights on the physics of flavour, revealing relations between masses and mixing angles, or specific textures in the quark mass matrix, which, if there, should be the consequence of a still undiscovered flavour symmetry.

Because of the confining property of strong interactions, the determination of quark masses and mixing angles requires a non-perturbative control of hadron dynamics. A major role, in this context, has been played by the numerical simulations of lattice QCD which have reached, in the last years,

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an accuracy unpaired by any other approach. With respect to other non-perturbative techniques, like QCD sum rules or the $1/N$ expansion, lattice calculations allow a better control of the systematic errors, which may be (and has been) systematically improved in time. Space-time discretization errors, which are inherent to lattice QCD calculations, have been reduced by the introduction of improved versions of the lattice QCD action [3]- [5], and the increase of computer power has allowed, in most of the cases, an extrapolation of the lattice results to the physical continuum limit. Another potential source of systematic error comes from the truncation of the perturbative expansion in the calculation of lattice renormalization constants (or mixing coefficients). For many physical quantities, like the quark masses or the B-parameters of four-fermion operators, this error has been reduced to a negligible amount by the use of non-perturbative renormalization techniques [6]- [8], which have proved to be a crucial ingredient in increasing the accuracy of the lattice determinations. Most likely, the largest source of uncertainty in lattice calculations is due, at present, by the use of the quenched approximation, derived by neglecting the effects of virtual quark loops. With the advent of the last generation of supercomputer, however, several unquenched calculations have been already performed, albeit with typically two flavours of dynamical quarks. In the last year, the first unquenched calculation of the b-quark mass has been performed [9], which also employs a non-perturbative renormalization technique. Other important unquenched results concern the calculation of the B-meson pseudoscalar decay constant f_B [10], which is relevant for the phenomenological studies of $B - \bar{B}$ mixing.

   In this talk some recent results of lattice QCD calculations which are relevant for the determination of the Standard Model fundamental parameters and for the phenomenology of particle physics are reviewed. These results concern the determination of the light (u, d and s) and bottom quark masses, the calculation of the heavy mesons decay constants, $f_{D_{(s)}}$ and $f_{B_{(s)}}$, and the calculation of the B_B and B_K parameters which control the amplitude of $B - \bar{B}$ and $K - \bar{K}$ mixings respectively. The results of a recent analysis of the CKM matrix and the unitarity triangle, which is based on the lattice calculations of the relevant hadronic matrix elements, are also presented. Finally, I will discuss the lattice prediction for the B_s^0-mesons lifetime difference which is found, within the Standard Model, to be possibly accessible to the experimental observation. For this review, some of the compilations of lattice results have been updated with new determinations which were not yet available at the time of the PIC20 conference.[2]

2   Quark Masses

Quark masses are fundamental parameters of the Standard Model which cannot be directly measured in the experiments because, unlike leptons, quarks are confined inside the hadrons. Being free parameters of the Standard Model lagrangian, quark masses cannot be computed on the basis of theoretical considerations only. Their values can be determined by comparing the result of a theoretical calculation of a given physical quantity, which depends on quark masses, with the corresponding experimental value. Typically, for instance, the pion and kaon masses are used to compute the values of the up-down and the strange quark masses, whereas the b-quark mass is determined by computing on the lattice the mass of the B or the Υ mesons. Different choices are all equivalent in principle, and the differences in the values of quark masses, obtained by using different hadron masses as input parameters, give an estimate of the systematic error involved in the calculation.

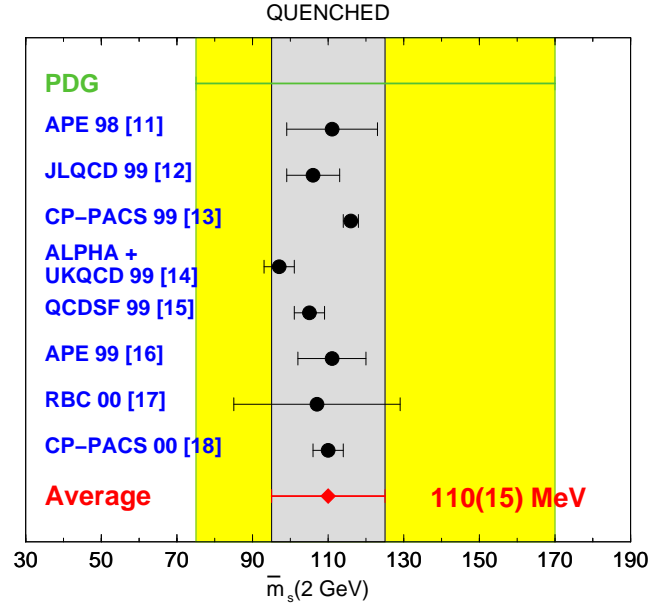[2]The transparencies of the talk at the PIC20 conference are available at the following URL site:
`http://www.lip.pt/pic20/Vittorio.Lubicz/`

Figure 1: Values of the strange quark mass, $\overline{m}_s(2\,\text{GeV})$, obtained from recent lattice calculations in the quenched approximation (circles). The upper line is the estimate of $\overline{m}_s$ quoted by the PDG [21], and the lowest point (diamond) represents the lattice (quenched) world average.

As any other free parameter of the Standard Model lagrangian, quark masses can be defined as effective couplings and, as such, are both renormalization scheme and scale dependent. A scheme commonly adopted for quark masses is the $\overline{\text{MS}}$ scheme, with a renormalization scale chosen in the short-distance region to make this quantity accessible to perturbative calculations. It is a common practice to quote the values of the light quark masses at the renormalization scale $\mu = 2$ GeV, whereas the heavy quark masses are usually quoted at the scale of the quark mass itself, e.g. $\overline{m}_b(\overline{m}_b)$.

Light Quark Masses

In the last few years a big effort has been devoted to compute on the lattice the value of the strange quark mass. The results of the more recent calculations obtained, in the quenched approximation, by using as experimental input the physical kaon mass, are shown in fig.1. All these results have been obtained adopting a non-perturbative renormalization technique, with the only exceptions of the two calculations by the CP-PACS collaboration [13, 18], in which the quark mass renormalization constant has been evaluated by using one-loop perturbation theory. Previous experience suggests that an additional uncertainty of the order of approximately 10%, due to the use of perturbation theory, should be added to the results quoted in refs. [13] and [18] to account for the corresponding systematic error. Besides that, the results presented in fig.1 are also affected by other sources of systematics which are different among the several calculations, but their total effect may be estimated to be of the order of few per cent. For instance, the APE calculations of refs. [11] and [16] do not involve the extrapolation to the continuum limit. In this case, the more extensive analysis of ref. [14] suggests that the value of the strange quark mass is underestimated by approximately 3%. Moreover, in the calculations of refs. [11] and [12] the conversion from the non-perturbative RI-MOM renormalization scheme to the $\overline{\text{MS}}$ scheme has been performed by using N^2LO perturbation theory [19], since at the time when these studies have been performed the N^3LO result of ref. [20]

was not available yet. In order to account for the difference between N^2LO and N^3LO, the results of refs. [11] and [12] should be decreased by approximately 3%.

Within the quenched approximation, the main source of systematic error which affects the determinations of the strange quark mass comes from the uncertainty in fixing the physical lattice scale (i.e. the lattice spacing). Because of the quenched approximation, different choices of the physical input, like the rho mass or the pion decay constant, lead to different estimates of the scale. This introduces an additional uncertainty, which is of the order of 10%, to the final estimate of the strange quark mass. By taking into account also this uncertainty, I quote as an average of the lattice results, within the quenched approximation,

$$\overline{m}_s(2\,\text{GeV})^{\text{QUEN}} = (110 \pm 15)\,\text{MeV}\,. \tag{1}$$

This value is also shown in fig.1, together with the lattice results and the average value of $\overline{m}_s$ quoted by the Particle Data Group [21] (PDG). Notice that the uncertainty affecting the lattice determination of $\overline{m}_s$ is approximately three times smaller than the one quoted by the PDG, although, in the former, the effect of the quenching approximation has not yet been taken into account.

The average value of the up and down quark masses, $\overline{m}_{u,d} \equiv (\overline{m}_u + \overline{m}_d)/2$, can be also computed in a similar way. It is convenient, however, to consider the ratio of the strange to the average up-down quark masses, because in this ratio many of the systematic uncertainties are expected to cancel out. On the lattice, this ratio is found to be in very good agreement with the value 24.4 ± 1.5 predicted by chiral perturbation theory [22]. By using this information and eq. (1), I obtain:

$$\overline{m}_{u,d}(2\,\text{GeV})^{\text{QUEN}} = (4.5 \pm 0.6)\,\text{MeV}\,. \tag{2}$$

The remaining uncertainty in the determination of the light quark masses is the effect of quenching. At present, unquenched studies of light quark masses have not yet reached the same degree of accuracy achieved in quenched calculations. In order to obtain an estimate of the quenching effect, it is thus convenient to compute directly the ratio between the quenched and unquenched values of the quark mass, both obtained by using the same discretized version of the QCD action, the same renormalization procedure and the same choice of physical inputs to fix the quark mass itself and the lattice scale. To date, the most extensive unquenched calculation of the strange quark mass (with two flavours of dynamical quarks) has been performed by CP-PACS [18]. They obtain the ratio $m_s^{\text{QUEN}}/m_s^{\text{UNQ}} = 1.25(8)$, suggesting a sizable decrease of the quark mass in the unquenched case. This result, however, is not confirmed by the other (although less accurate) unquenched studies, by SESAM [23], APE [24] and MILC [25], which find a decrease in the unquenched case rather of the order of 10%. Given the present situation, I believe that, in order to quote a final estimate of the lattice results, it is appropriate to include the quenching error as an additional systematic uncertainty in eqs. (1) and (2), rather than varying the central values. Assuming this error to be of the order of 20 MeV in the case of the strange mass, I obtain:

$$\overline{m}_s(2\,\text{GeV}) = (110 \pm 25)\,\text{MeV} \tag{3}$$

and

$$\overline{m}_{u,d}(2\,\text{GeV}) = (4.5 \pm 1.0)\,\text{MeV}\,. \tag{4}$$

The b-quark Mass

Since the b-quark mass is larger than typical values of the ultraviolet cutoff in present lattice calculations ($a^{-1} \sim 3$ GeV), the b-quark cannot be directly simulated on the lattice. However, the b mass

is also larger than the typical energy scale of strong interactions, and the heavy degrees of freedom of the b-quark can be integrated out. Beauty hadrons may be therefore simulated on the lattice in the framework of a low-energy effective theory. Indeed, in the past years, many lattice simulations of the Heavy Quark Effective Theory (HQET) and Non-Relativistic QCD (NRQCD) have been performed.

Within the effective theory, the mass of a B-meson, M_B, is related to the pole mass of the b-quark through the relation:

$$M_B = m_b^{\text{pole}} + \varepsilon - \delta m \tag{5}$$

where ε is the so-called binding energy, which can be computed by a numerical simulation in the effective theory, and δm is the residual mass, generated, in the effective theory, by radiative corrections. The perturbative calculation of the residual mass and the non-perturbative calculation of the binding energy thus allow a determination of the pole mass of the b-quark. The result can be then translated into the $\overline{\text{MS}}$ mass, $\overline{m}_b$, by using again perturbation theory.

An important observation, concerning this procedure, is that the binding energy ε is not a physical quantity, and it is affected indeed by a power divergence proportional to the inverse lattice spacing, $1/a$. This divergence is canceled by a similar singularity in the residual mass δm. Moreover, the pole mass m_b^{pole}, and consequently δm, are also affected by a renormalon singularity, which introduces in their definitions an uncertainty of the order of Λ_{QCD}. This singularity is then canceled by the perturbative series relating the pole mass and the $\overline{\text{MS}}$ mass. As a result, the $\overline{\text{MS}}$ mass, $\overline{m}_b$, is a finite, well defined, short-distance quantity. In the actual calculation, however, since the residual mass is computed up to a finite order in perturbation theory, only a partial cancellation of both the power divergence and the renormalon singularity may occur. For this reason, it is crucial in this calculation to compute the residual mass δm up to the highest possible order in perturbation theory.

At present, the most accurate determination of δm has been obtained in the framework of HQET. The two-loop analytical calculation has been performed in ref. [26]. In the quenched case, also the three-loop coefficient of the perturbative expansion has been evaluated [27], by using a numerical technique called numerical stochastic perturbation theory. The result is confirmed by the (less accurate) determination of ref. [28], obtained with a completely different approach, by fitting the results of small coupling Monte Carlo calculations. These combined theoretical efforts allow a determination of the b-quark mass which is accurate, in the quenched approximation, up to the N^{3}LO. Two independent results have been obtained so far:

$$\overline{m}_b^{\text{QUEN}} = (4.30 \pm 0.05 \pm 0.05)\ \text{GeV} \qquad\qquad [29]$$

$$\overline{m}_b^{\text{QUEN}} = (4.34 \pm 0.03 \pm 0.06)\ \text{GeV} \qquad\qquad [30] \tag{6}$$

nicely in agreement within each other. The last error in eqs. (6) represents the residual uncertainty due to the neglecting of higher orders in perturbation theory. This uncertainty was estimated to be approximately 200 MeV and 100 MeV at the NLO [31] and N^{2}LO [26] respectively. By using NRQCD, results compatible with those in eqs. (6) have been obtained. However, being only accurate at NLO, they are affected by a larger theoretical uncertainty.

The first unquenched calculation of the b-quark mass has been performed this year [9]. The result, which is accurate at the N^{2}LO, since the third coefficient of the perturbative expansion of δm is yet unknown in the unquenched case, reads:

$$\overline{m}_b = (4.26 \pm 0.06 \pm 0.07)\ \text{GeV} . \tag{7}$$

This estimate represents to date the most accurate determination of the b-quark mass from lattice QCD calculations. Remarkably, the relative uncertainty is reduced at the level of 2%.
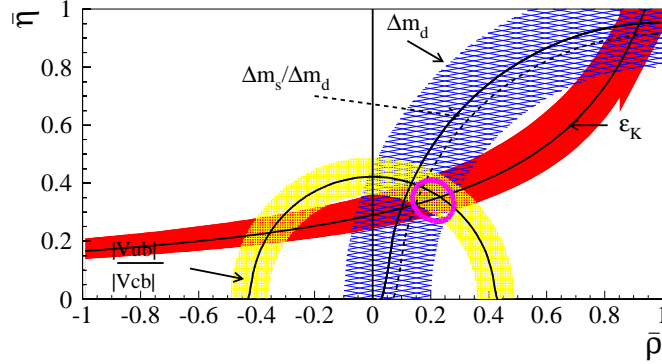
Figure 2: The allowed region for $\overline{\rho}$ and $\overline{\eta}$ is shown together with the uncertainty bands (at 68% probability for $|V_{ub}/V_{cb}|$, $|\varepsilon_K|$, Δm_d and the limit on $\Delta m_d/\Delta m_s$. The picture is taken from ref. [32].

3 $B-\bar{B}$ mixing, $K-\bar{K}$ mixing and the Unitarity Triangle from Lattice QCD

One of the most important test of the Standard Model, and a powerful tool for the search of new physics, is the analysis of the CKM unitarity triangle. At present, this analysis is based on the study of four different constraints, coming from the experimental determinations of the following quantities: the ratio $|V_{ub}/V_{cb}|$, which determines the relative rate of $b \to u$ and $b \to c$ transitions in semileptonic decays, the neutral B-meson mass differences, Δm_d and Δm_s, which control the frequencies of $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ oscillations, and the parameter ε_K, which defines the extent of indirect CP violation in kaon decays. Within the Standard Model, all these quantities are functions of the four Wolfenstein parameters of the CKM matrix, A, λ, $\overline{\rho}$ and $\overline{\eta}$:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \frac{\lambda}{1 - \lambda^2/2} \sqrt{\overline{\rho}^2 + \overline{\eta}^2} \, , \tag{8}$$

$$\Delta m_d = C_B \, m_{B_d} \, f_{B_d}^2 \hat{B}_{B_d} \, A^2 \lambda^6 \left[(1 - \overline{\rho})^2 + \overline{\eta}^2 \right] , \tag{9}$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d}}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s}} \, \lambda^2 \left[(1 - \overline{\rho})^2 + \overline{\eta}^2 \right] , \tag{10}$$

$$|\varepsilon_K| = C_K \, \hat{B}_K \, A^2 \lambda^6 \, \overline{\eta} \left[A^2 \lambda^4 \, (1 - \overline{\rho}) \, F_{tt} + F_{tc} \right] . \tag{11}$$

The coefficients $C_{B,K}$ and $F_{tt,tc}$ are known quantities, while $f_{B_{d,s}}$, $\hat{B}_{B_{d,s}}$ and $\hat{B}_K$ are the pseudoscalar decay constants and B-parameters which encode, in the above equations, the non-perturbative effects of the strong interactions. For each quantity in eqs. (8)-(11), the comparison of the theoretical expression with the corresponding experimental measurement defines a curve in the $\overline{\rho}$-$\overline{\eta}$ plane which, in reality, because of the experimental and theoretical uncertainties, becomes a band in this plane. Consistency of the Standard Model requires that the four bands, corresponding to the different constraints, all intersect each other in the same region, which in turn defines a set of allowed values for the $\overline{\rho}$ and $\overline{\eta}$ parameters. A recent analysis based on this approach has been performed in ref. [32], and
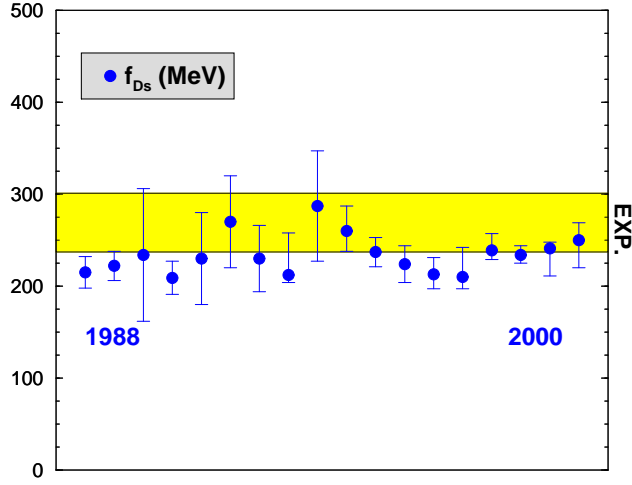
Figure 3: Quenched lattice results for the pseudoscalar decay constant f_{D_s} plotted as a function of time. The band indicates the present experimental average.

the results are illustrated in fig. 2. Quantitative estimates of $\overline{\rho}$ and $\overline{\eta}$, and the angles of the unitarity triangle, will be given at the end of this section.

The main theoretical issue in the analysis of the CKM triangle is the non-perturbative calculation of the pseudoscalar decay constants and B-parameters entering in eqs. (8)-(11). Lattice QCD provides the optimal tool to perform such calculations and, in the following, I will review the most recent results of the lattice studies.

$B-\bar{B}$ **Mixing: Decay Constants and B-parameters**

The reliability of lattice calculations in computing the values of the pseudoscalar decay constants of D and B mesons has been recently supported by the experimental measurement of f_{D_s}, a quantity which has been computed on the lattice since many years. In 1988, one of the first lattice calculation of f_{D_s} predicted, in the quenched approximation, the value $f_{D_s} = (215 \pm 17)$ MeV [33]. Subsequently, as shown in fig. 3, the lattice predictions for f_{D_s} have been always very stable in time. All lattice results obtained in the quenched approximation, in a period which extends over approximately 12 years, are presented in the figure as a function of the time, and the present experimental average is also shown for comparison. From the most recent lattice determinations of f_{D_S}, I obtain the quenched average:

$$f_{D_s}^{\text{QUEN}} = (235 \pm 20) \text{ MeV} \tag{12}$$

Unquenched calculations of f_{D_S} have been performed by the MILC [10] and CP-PACS [34] collaborations. They find an increase of the decay constant, in the unquenched case, of approximately 10%, namely $f_{D_s}^{\text{UNQ}}/f_{D_s}^{\text{QUEN}} = 1.09(14)$ (MILC) [3] and $1.07(5)$ (CP-PACS). This correction can be then included in eq. (12) to obtain, as a final estimate of f_{D_S} from lattice calculations, the value:

$$f_{D_s} = (250 \pm 25) \text{ MeV} \tag{13}$$

This prediction is in remarkable agreement with the present experimental average, $f_{D_s} = (271^{+30}_{-34})$ MeV [35].

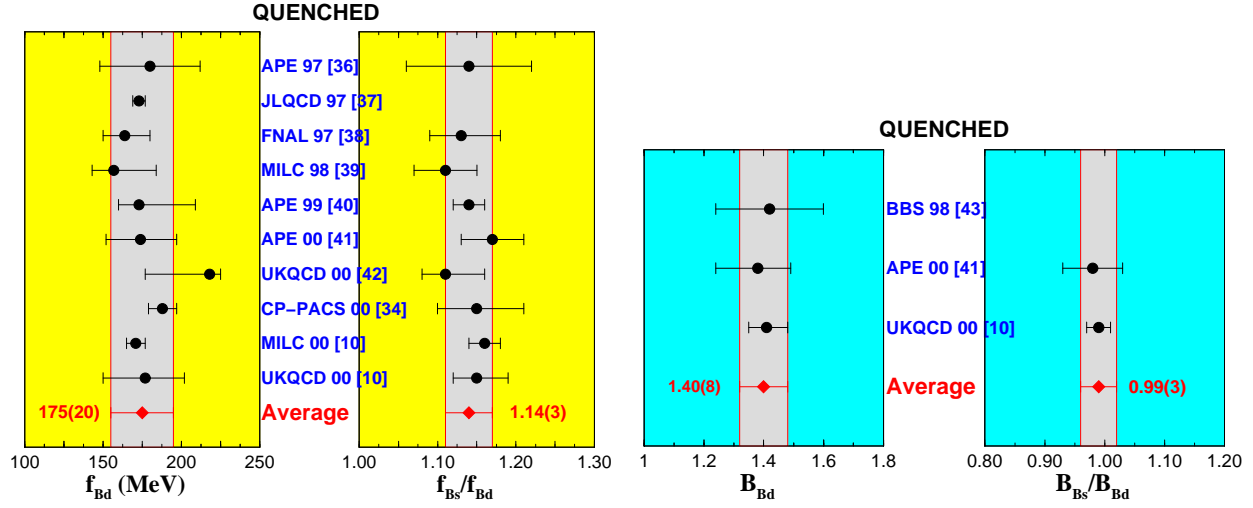[3]The error is my estimate based on the MILC results.

Figure 4: Values of f_{B_d}, f_{B_s}/f_{B_d}, $\hat{B}_{B_d}$ and $\hat{B}_{B_s}/\hat{B}_{B_d}$ obtained, in the quenched approximation, from recent lattice calculations.

The calculation of the B-meson decay constant, f_B, which is not yet measured in the experiments, has been also the subject of intense activity of lattice QCD simulations. The most recent results for f_{B_d} and for the ratio f_{B_s}/f_{B_d}, obtained in the quenched approximation [10,34], [36]- [42] are shown in fig. 4 (left). Results for the decay constants have been also obtained by using NRQCD [44]- [46]. In this case, however, the predictions of different calculations are incompatible among each other, possibly signalling the presence of underestimated systematic effects. For this reason, the NRQCD results for the B-meson decay constants and for the B-parameters have not been included in the final lattice averages. From the results of fig. 4, I obtain the estimates:

$$f_{B_d}^{\text{QUEN}} = (175 \pm 20)\,\text{MeV} \qquad , \qquad (f_{B_s}/f_{B_d})^{\text{QUEN}} = 1.14 \pm 0.03 \qquad (14)$$

which, for comparison, are also shown in the figure.

In the vacuum saturation approximation (VSA), the pseudoscalar decay constants define the values of the matrix elements of the four-fermion operators which are relevant for $B-\bar{B}$ mixing. Deviations from the VSA are expressed, instead, by the B_B parameters. The recent quenched lattice determinations of $\hat{B}_{B_d}$ and $\hat{B}_{B_s}/\hat{B}_{B_d}$ (the hat denoting the renormalization group invariant definition of these parameters) are shown in fig. 4 (right). Although only few groups have performed such a calculation, the results are in very good agreement among each other, so that one can derive the rather accurate averages shown in the figure, namely $\hat{B}_{B_d}^{\text{QUEN}} = 1.40 \pm 0.08$ and $(\hat{B}_{B_s}/\hat{B}_{B_d})^{\text{QUEN}} = 0.99 \pm 0.03$.

A source of uncertainty, in the lattice studies of B-physics, is introduced by the necessity to extrapolate the results, reliably computed in the charm mass region, where discretization effects are under control, to the b-quark mass. In the case of the B-parameters, this uncertainty can be estimated, and partially reduced, by combining the relativistic results of fig. 4 with the prediction obtained in the infinite mass limit, $\hat{B}_{B_d}^{\text{HQET}} = 1.29 \pm 0.08 \pm 0.06$ [47]. From this analysis, at the value of the B-meson mass, one gets $\hat{B}_{B_d} \simeq 1.3 \pm 0.1$. Since this estimate combines the results of different theories, affected by different systematic uncertainties, I prefer to quote, as a final central value for $\hat{B}_{B_d}$, in the quenched approximation, the average between the relativistic result and the one obtained combining with HQET, including in the systematic error the differences between the two determinations. As far

as the ratio $\hat{B}_{B_s}/\hat{B}_{B_d}$ is concerned, this is only marginally affected by the extrapolation, since $1/M$ corrections are negligible in this case. One then obtains:

$$\hat{B}_{B_d}^{\text{QUEN}} = 1.36 \pm 0.10 \qquad , \qquad (\hat{B}_{B_s}/\hat{B}_{B_d})^{\text{QUEN}} = 0.99 \pm 0.03 \qquad (15)$$

The above estimates should be improved to account for the effect of the quenched approximation. Unquenched calculations of the pseudoscalar decay constants of B-mesons have been performed by the MILC [10] and CP-PACS [34]collaborations. They find the ratio $f_{B_d}^{\text{UNQ}}/f_{B_d}^{\text{QUEN}} = 1.12^{+0.16}_{-0.11}$ (MILC) and 1.11 ± 0.06 (CP-PACS), in very good agreement within each other. For the ratio f_{B_s}/f_{B_d}, no significant difference between quenched and unquenched determinations has been observed by both MILC and CP-PACS, so that the quenching error on this quantity should be practically negligible. Starting from eq. (14), I then obtain, as final lattice estimates of the decay constants, the values:

$$f_{B_d} = (200 \pm 25) \text{ MeV} \qquad , \qquad f_{B_s}/f_{B_d} = 1.14 \pm 0.03 \qquad (16)$$

Unfortunately, unquenched calculations of the B_B parameters have not been performed to date. For these quantities, theoretical estimates based on quenched chiral perturbation theory [48] suggest that the quenching error may be of the order of 10% at most. In the lack of a direct unquenched calculation, I rely on these conservative estimates, and include this uncertainty in the systematic error, obtaining:

$$\hat{B}_{B_d} = 1.36 \pm 0.17 \qquad , \qquad \hat{B}_{B_s}/\hat{B}_{B_d} = 0.99 \pm 0.10 \qquad (17)$$

Finally, combining eqs. (16) and (17), the lattice estimates for the two relevant parameters entering the theoretical expressions of Δm_d and $\Delta m_d/\Delta m_s$ are derived:

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = (230 \pm 35) \text{ MeV} \qquad (18)$$

and

$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.14 \pm 0.06 \qquad (19)$$

In the standard analysis of the unitarity triangle, four constraints are used to determine the values of only two parameters, $\overline{\rho}$ and $\overline{\eta}$. Therefore, an interesting possibility consists in relaxing, in turn, the theoretical estimate of one of the hadronic quantities relevant for the analysis, determining its value together with the values of the CKM parameters. In this way, a 68% probability interval has been obtained for $f_{B_d}\sqrt{\hat{B}_{B_d}}$ in ref. [32]:

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = (229 \pm 12) \text{ MeV} \qquad (20)$$

which is nicely consistent with the lattice determination of eq. (18). Notice also that, at present, the analysis of the unitarity triangle, based on the lattice determinations of ξ and $\hat{B}_K$, provides an estimate of $f_{B_d}\sqrt{\hat{B}_{B_d}}$ which is more accurate than the direct theoretical determination. Therefore, a strong effort should be put in lattice calculations to improve this estimate at the level of accuracy of 10% or better.

$K - \bar{K}$ **Mixing and the** B_K **parameter**

The B_K parameter, which encodes the effects of strong interactions in the hadronic matrix element relevant for $K - \bar{K}$ mixing, has been extensively studied in lattice QCD simulations. The two most accurate determinations, within the quenched approximation, have been obtained by using staggered fermions, and they both involve an extrapolation to the continuum limit. The two results, expressed in terms of the renormalization group invariant definition of the parameter, read:

$$\hat{B}_K^{\text{QUEN}} = 0.86 \pm 0.04 \qquad [49]$$

$$\hat{B}_K^{\text{QUEN}} = 0.87 \pm 0.06 \qquad [50] \qquad (21)$$

in good agreement within each other. Several lattice calculations of B_K have been also performed by using Wilson fermions. The results are generally consistent with those in eqs. (21), but with larger systematic errors. This is due to the explicit breaking of chiral symmetry in the Wilson action which requires, in the definition of the renormalized operator, a delicate non-perturbative subtraction of the operators with different chirality. A simple prescription to avoid the subtraction has been recently proposed in ref. [51]. If effective in practice, this procedure should allow to obtain, also with Wilson fermions, more accurate determinations of the kaon B-parameter.

The quenching effect on B_K has been estimated in ref. [48] by using quenched chiral perturbation theory, and a direct unquenched calculation has been also performed in [52], at a fixed value of the lattice spacing. Both studies estimate the quenching effect to be approximately 5%. In order to take also into account the uncertainty on this estimate, the systematic error induced by the use of degenerate quark masses in the calculations of $\hat{B}_K$ [48] and the discretization effects in the unquenched case, the total uncertainty is (conservatively) increased to 15%. In this way, by using eqs. (21), one obtains:

$$\hat{B}_K = 0.87 \pm 0.14 \,. \qquad (22)$$

As for the case of $B - \bar{B}$ mixing, the lattice estimate of $\hat{B}_K$ is in perfect agreement with the result of the overconstrained fit of the Standard Model [32]:

$$\hat{B}_K = 0.89^{+0.21}_{-0.15} \,. \qquad (23)$$

This agreement provides additional evidence of the good level of accuracy reached, at present, by lattice calculations.

The Unitarity Triangle: a "Lattice-based" Analysis

The lattice determinations of $\hat{B}_K$, $f_{B_d}\sqrt{\hat{B}_{B_d}}$ and ξ provide the values of the input hadronic parameters entering the analysis of the unitarity triangle. One of these analysis, in which special attention has been devoted to the determination of the related theoretical uncertainties, has been performed in ref. [32]. The purpose of this study is to infer regions of the parameter space in which the values of the CKM parameters lie with given probabilities. In ref. [32], at 68% probability, it is found

$$\overline{\rho} = 0.21 \pm 0.04 \,, \qquad \overline{\eta} = 0.34 \pm 0.04 \qquad (24)$$

which imply, for the angles of the unitarity triangle, the values

$$\sin(2\beta) = 0.72 \pm 0.07 \,, \qquad \sin(2\beta) = -0.28 \pm 0.27 \,, \qquad \gamma = (59 \pm 7)^o \,. \qquad (25)$$

The allowed region for the $\overline{\rho}$ and $\overline{\eta}$ parameters is also shown in fig. 2. As can be observed from the figure, the Standard Model predictions are fully consistent with the experimental measurements of semileptonic b-decays, $B - \bar{B}$ mixing and $K - \bar{K}$ mixing, within the present level of theoretical and experimental accuracy. Moreover, the inferred value of $\sin(2\beta)$ is consistent with the direct measurement from $J/\psi K_S$ events, by LEP, CDF, BaBar and Belle, which give the average $\sin(2\beta) = 0.52 \pm 0.22$ [32]. Although the direct measurement has not yet reached a significant accuracy, important progresses are expected to come soon from the B-factories, thus allowing a crucial test of consistency with the unitarity triangle determination.

4 The B_s^0-mesons Lifetime Difference

In the Standard Model, the lifetime difference between the *short* and the *long* B_s^0-mesons is expected to be rather large, and possibly within reach for being measurable in the near future. In ref. [53], the following experimental bound has been obtained:

$$\left(\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}}\right)^{\text{EXP}} < 0.31 \qquad \text{at } 95\% \, CL \tag{26}$$

Theoretically, the prediction of $(\Delta\Gamma_{B_s}/\Gamma_{B_s})$ relies on the use of the operator product expansion, where the large scale is provided in this context by the heavy quark mass. The theoretical expression for the width difference can be schematically written in the form:

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = K \left(G(z) - G_S(z)\,\mathcal{R} + \delta_{1/m}\right) \tag{27}$$

where K is a known factor, and $G(z)$ and $G_S(z)$, with $z = m_c^2/m_b^2$, are Wilson coefficients which have been computed by including QCD radiative corrections at the NLO [54]. The factor $\delta_{1/m}$ contains the subleading contribution in the $1/m_b$ expansion [55]. The non-perturbative strong interaction effects, in eq. (27), are encoded in the ratio of the matrix elements of two four fermion operators,

$$\mathcal{R} = \frac{\langle \bar{B}_s^0 | Q_S | B_s^0 \rangle}{\langle \bar{B}_s^0 | Q_L | B_s^0 \rangle} \tag{28}$$

where $Q_L = \bar{b}\gamma_\mu(1 - \gamma_5)s\,\bar{b}\gamma_\mu(1 - \gamma_5)s$ and $Q_S = \bar{b}(1 - \gamma_5)s\,\bar{b}(1 - \gamma_5)s$.
At present, quenched lattice calculations of the ratio $\mathcal{R}$ have been performed in refs. [56] and [57], the latter by using the NRQCD effective theory. The results of ref. [57] have been subsequently corrected in ref. [58], and the two results are now nicely in agreement:

$$\mathcal{R} = -0.93 \pm 0.03^{+0.00}_{-0.01} \qquad\qquad [56]$$

$$\mathcal{R} = -0.91 \pm 0.03 \pm 0.12 \qquad\qquad [58] \tag{29}$$

For the B_s^0-mesons lifetime difference, these measurements imply the prediction [56]:

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = (4.7 \pm 1.5 \pm 1.6) \times 10^{-2} \tag{30}$$

which is small but possibly accessible to the experimental observation. It should be noted that, despite the agreement between the results in eq. (29), a much larger prediction for the width difference

has been obtained in ref. [58]. The reason is that, in order to get a prediction for $\Delta\Gamma_{B_s}$ a quite large unquenched estimate of f_{B_s} (namely $f_{B_s} = (245 \pm 30)$ MeV, which is at the upper bound of the present lattice average) has been combined in ref. [58] with the quenched determination (29) of the matrix elements. As shown in ref. [56], however, a more accurate theoretical estimate of $\Delta\Gamma_{B_s}/\Gamma_{B_s}$ can be obtained in terms of the hadronic parameter ξ which, compared to f_{B_s}, is much less affected by theoretical uncertainties. In this way, the prediction (30) has been derived.

5 Conclusions

The results of lattice QCD calculations are playing a crucial role in determining the values of the Standard Model free parameters. The lattice determinations of quark masses have reached at present a high-level of statistical and systematical accuracy, which is of the order of 2% in the case of the b-quark mass. For light quarks the accuracy is at the level of 10%, although the effect of the quenching approximation still requires, in this case, further investigations. The lattice studies of $B - \bar{B}$ and $K - \bar{K}$ mixings are of crucial importance for the determinations of the CKM matrix elements and for the analysis of the unitarity triangle. Moreover, the comparison between the lattice predictions and the overconstrained fits of Standard Model, whenever available ($f_B\sqrt{B_B}$ and B_K), reveals an high degree of consistency.

Acknowledgments

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